2.2 Laws of Logarithms

Question Paper

Course	CIE AS Maths
Section	2. Logs & Exponentials
Topic	2.2 Laws of Logarithms
Difficulty	Very Hard

Time allowed: 50

Score: /40

Percentage: /100

Question 1a

(a) Evaluate

$$4\log_3 729 + 3\log_2 64^2 - 3\log 100 + \ln e^6$$

[2 marks]

Question 1b

(b) Evaluate

$$\frac{1}{2}\ln 196 + \frac{1}{3}\ln 125 + \frac{1}{4}\ln 81 + \frac{1}{5}\ln 32$$

giving your answer in the form $\ln q$.

[3 marks]

Question 2

Solve the equation

$$2 \times 5^{2x+1} + 21 = 41 \times 5^x,$$

giving your answers in the form $\log_a b$, where a and b are rational numbers to be found.

[4 marks]

Question 3a

Solve the following equations, giving your answers correct to 3 significant figures.

(a) $8e^{3x^2-1} = 12$

[3 marks]

Question 3b

(b) $e^{3x} - 42 = 2e^{x}(6e^{x} - 7)$

[3 marks]

Question 4

Show that

$$2\log_3 x + \log_3(x^2 - 1) - 2\log_3(x + 1) \equiv \log_3 \frac{x^2(x-1)}{(x+1)}.$$

[3 marks]

Question 5

Write the following as a single logarithm

$$2\log_p(x + 1) + 3\log_p(x - 1) - \log_p(x^2 - 1).$$

[3 marks]

Question 6

On the same axes, sketch the graphs of $y = e^{2x}$ and $y = \frac{1}{2}\ln x$.

On each graph, label any points where the graph intersects the coordinate axes.

Write down the equations of any asymptotes for each graph.

Explain the significance of the line $y = x$.

[5 marks]

Question 7

Show that $4 - \ln 16$ can be written in the form $4\ln(\frac{e}{2})$.

[3 marks]

Question 8

A triangle is drawn inside a circle such that one side of the triangle is the diameter and all three vertices of the triangle lie on the circumference.
The radius of the circle is $(3 \ln 2)$ cm.
The two smallest angles in the triangle are α and β respectively where $\beta = 2\alpha$.
Find all three sides of the triangle, giving your answers in the form $a\ln 2$.

[5 marks]

Question 9

How many real solutions does the equation have? Justify your answer.

$$3\log_x(x+1) = \ln e^3$$

[3 marks]

Question 10

Without using a calculator, show that

$$\log_4 8 = \log_9 27\,.$$

[3 marks]